Lucas Paciola and the Art of Double Entry Book Keeping

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Abstract

We briefly discuss the evolution of mathematical textbooks ranging from Egypt and Greece in Ancient times to the slightly more modern writings of Fibonacci and Pacioli. We will give one specific example of this evolution by tracing the method of multiplying multiple digit numbers together in the various textbooks.

Then we will consider more specifically Pacioli's treatise on book keeping.

Lastly we will, again briefly, discuss Pacioli himself.

Contents

1 Introduction

This paper is the direct outcome of the following three paragraphs, found in the introduction to a treatise on the art of double entry bookkeeping. I include them in full and note that these words, true in 1914 when they were published, still largely hold true today.

> *Mr. John P. Young, Editor of the San Francisco Chronicle, . . . showed that Rome in Cicero's time was dependent upon the independent verification of accounts and statements thereof by one skilled in accountancy. The familiarity with which he mentions the accountant would seem to indicate that his place in the Roman social organization was well established.*
>
> *However, after the recorded utterances of Cicero the historian finds in the pages of history no further mention of those individuals acknowledged to be skilled in accounts, which we are pleased to call accountants, until the writings of Pacioli in 1494 and Stevin in 1604[Geijsbeek, 1914].*

Fra Luca Pacioli, whose guide to double entry bookkeeping was published almost as an afterthought in his textbook *Summa de arithmetica, geometria, proportioni et proportionalita*[1], is today known as the Father of Accounting. His textbook was one of the first books published using the novel invention of movable type in 1494, and was reprinted in 1523. The first part of the book is a distillation of the mathematics taught to the children of merchants, and was probably based in large part on the writings of predecessors such as Leonardo of Pisa's *Liber Abaci*[2]. After covering the standard materials taught in mathematics classes, including the use of Hindu/Arabic numerals[3] which inspired the notion of "counting in the hands"[4], and "$\tilde{p}$" and "$\tilde{m}$" for the plus and minus symbols, respectively[5], Pacioli turns to the study of accounting methods. Pacioli describes these accounting methods, as used by the merchants of Venice, in careful detail; from a description of the different ledgers and journals that must be kept to explanations of how and where and when debits and credits should be entered.

[1]For brevity's sake, we will abbreviate the long title *Summa de arithmetica, geometria, proportioni et proportionalita* as *Summa* throughout this paper.

[2]Leonardo of Pisa is today better known as Fibonacci, and I will refer to him as such.

[3]Fibonacci had introduced these numerals in his *Liber Abaci* in 1202, but roman numerals were still widely used.

[4]Counting in the hands allows you to count to 99 using one hand or 9999 using two hands, but relies on the Hindu/Arabic notion of place value. See also Figure 1 on page 4.

[5]Until then there was no standard notation for plus and minus, and different authors used different notations. Pacioli's notation became standard among Italian Renaissance mathematicians.

In this paper, we will first briefly consider the historical background and the evolution of mathematics textbooks using as examples the Rhind Papyrus written by Ahmose[6], Euclid's *Elements*[7], Fibonacci's *Liber Abaci*, and Pacioli's *Summa*, and the specific arithmetic operation of multiplying multi-digit numbers. Next we will focus explicitly on Pacioli's *Summa*, specifically the section devoted to the keeping of mercantile records, and consider what this book can tell us about the bookkeeping used by Venetian merchants.

2 The Art of Mathematical Textbooks

Euclid's *Elements* has the distinction of being the oldest, continuously used mathematical textbook – even today the definitions, theorems, and proofs form a basis for Geometry as taught in the public high schools. The *Elements* were written – or rather collected – by Euclid around 300 BC while he lived in Alexandria. Some of the results were the work of earlier Greek mathematicians, including Pythagorus[8], Hippocrates of Chios[9], Theatetus of Athens, and Edoxus of Cnidos, but Euclid's contribution was the clear and comprehensible exposition of the log behind the proofs. Given that two millennia have passed, it is impossible to know which parts were written by whom.

Unlike its predecessors, the *Elements*[10] were studied extensively, frequently recopied, often translated and re-translated, and so passed down through generations, libraries, universities, and across continents. Using the translations and redactions of Boethius, circa AD 500, the *Elements* formed a core part of the Quadrivium[11] which combined with the Trivium[12] as the seven liberal arts that were first taught in monasteries and cathedral schools, and later in universities as the latter became established.

[6] The Rhind Papyrus dates back to 1650 BC, but was named for Henry Rhind who purchased the papyrus in AD1858, which was donated to the British museum in 1864 by Henry Rhind's estate[Robins and Shute, 1987]. We will strive not to comment on the irony of a major mathematical paper being named for "some bloke who bought it at a bazaar". We will fail, but we will continue to strive.

[7] Written circa 300 BC by Euclid. See [Heath, 1956] for one of many translations, since the *Elements* are still very popular.

[8] Pythagorus is today famed for his proof about the relationship between the lengths of the sides of a right-angled triangle, but actually founded an entire cult of Mathematics named the Pythagoreans.

[9] Not to be confused with his contemporary Hippocrates of Kos, who is famous for his achievements in medicine.

[10] The *Elements* are actually a series of 13 books (an archaic form of chapters) primarily concerned with the study of geometric forms, although there are some attempts to recast arithmetic and algebraic problems in the form of geometry. See [Heath, 1956] for a complete online translation from the original Greek.

[11] The Quadrivium covered the study of arithmetic, geometry, astronomy, and number theory

[12] The Trivium covered the study of grammar, logic, and rhetoric.

2.1 Geometry versus Arithmetic

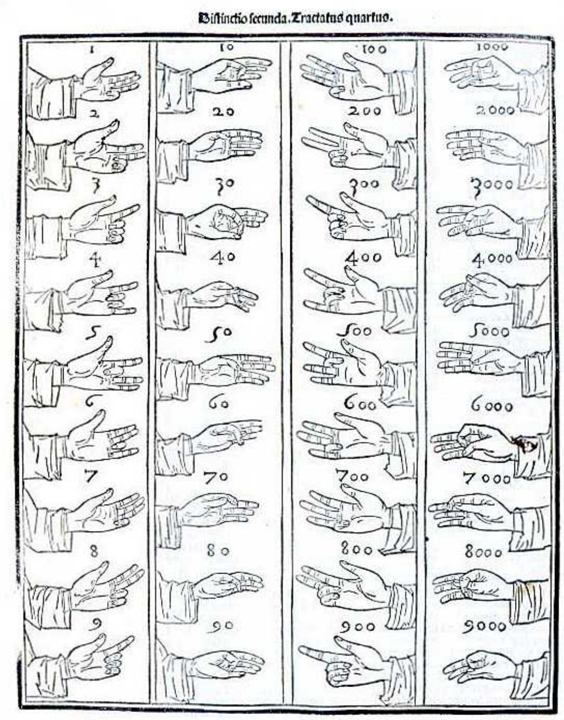


Figure 1: Counting in the hands using both hands, the left hand used for the smaller numbers. I have personally found that the easiest way to count in the hand is to press the hand – once the fingers are in the right position – lightly against a table or other surface, since my fingers routinely struggle to stay in position. Woodcut from Pacioli's Summa [Pacioli, 1494].

Each book[13] in the *Elements* includes definitions, axioms and theorems, as well as the proof of theorems. By studying the proofs students developed an understanding of logic that was particularly susceptible to being tested using only simple tools; a straight edge, a pencil, a compass[14]. What was lacking, however, was arithmetic; it was possible to design an arch that was twice as tall as it was wide, and then the relative widths and lengths of all the parts were known, but specific lengths still had to be calculated. In the Quadrivium the writings of masters other than Euclid were studied, but there was no single text that covered the fields of arithmetic and of what we call algebra[15] in modern days. Hence the need for additional textbooks that covered arithmetic and algebra.

One such early textbook was Fibonacci's *Liber Abaci*, first published in 1202[16] with a second edition in 1228. Fibonacci was born circa 1170 to the merchant class, and traveled extensively trading and studying mathematics before settling down as a citizen of the city-state of Pisa[17]. Fibonacci clearly mastered Euclid's *Elements*, and in fact modelled his own textbook on the *Elements*; solutions of complicated problems are not only shown,

[13]In modern times, we would call these books chapters.

[14]And a simple compass can be created using a pencil, a pin, and a string.

[15]While they did not have a specific name for algebra, they did already fully understand the notion of solving for the unknown in a problem. Lacking the terminology we have today, they just took many more words, and many more pages, describing what we can today write in a simple mathematical sentence.

[16]Since this predates movable type by several centuries, I should note that this version was available only as a hand-copied manuscript. No original copies of the 1202 version exist, but the 1228 version was later reprinted and eventually translated by Laurence Sigler [Sigler, 2002].

[17]Based on historical overview in the introduction of [Sigler, 2002].

but argued[18] to be correct. Moreover, in introducing the use of Hindu/Arabic numerals, Fibonacci showed the practical value of the new system[19] by using the new system to solve mathematical problems particularly suited to the mercantile class such as the conversion of weights and measures, the calculation of interest, and the changing of moneys as every major city had its own mint with its own coins.

As a diversionary sidenote, *Liber Abaci* translates roughly to "Freedom from the Abacus", and it was Fibonacci's intent to teach his students how to perform arithmetic without the use of a cumbersome abacus, by using a number system that includes place value[20], and a method of keeping numbers "in the hand" quite literally by showing students how to represent any number from 0 to 99 using one hand by representing the tens digit with the thumb and forefinger, and the ones digit with the middle, ring, and pinky finger as we can see in Figure 1 on page 4. Once Fibonacci moves past the rules of arithmetic – including the multiplication of two, three, and four digit numbers – he turns to the problem of solving for the unknown, the "thing" or "cosa" as it is called in Italian, for many different types of unknowns.

Liber Abaci was very much – like Euclid's *Elements* – a model mathematics textbook in that it was copied, borrowed from, and adapted in many different ways. Some of Fibonacci's topics, such as counting in the hand, remained unchanged for three or more centuries until they were again repeated by Pacioli. In other cases, Fibonacci's solutions were superseded by Pacioli's more modern solutions. One such example is the case of multidigit multiplication, multiplying for instance 982×31.[21] The difference in calculating 982×31 changes rapidly from the Egyptian method, to the Roman adaptation, and then to the methods described by Fibonacci and Pacioli, and we will consider these three methods and how both the method and its description evolved as an example of the evolution of these mathematical texbooks.

When considering the difference in the three presented solutions, the most striking

[18] A lot of the arithmetic and algebraic structure needed to provide an algebraic proof had not been invented yet, but the arguments are usually convincing. Wordy in the extreme, but eventually convincing. See the lengthy translations in [Sigler, 2002].

[19] The old system then in common use was the use of Roman numerals. The Roman system does not properly contain the concept of place value.

[20] The prior system, using roman numerals, would use I and V for the ones digits, X and L for the tens place, C and D for the hundreds, and so on. The hindu/arabic system used the same digits but relied on the place value to indicate whether the 1 represent a single unit, or ten units, or a 100 units, etc.

[21] Our modern calculator tells us that $982 \times 31 = 30442$, but even Leonardo da Vinci – student of Pacioli – had not gotten around to inventing a calculator yet, and Charles Babbage is sadly post period. So we shall calculate this number the old-fashioned way(s).

difference is that Pacioli is the first to abstract out the method from the example; previous authors dating back all the way to Ahmose, author of the Egyptian Rhind Papyrus[22], explain their method by giving a specific example and expecting the reader to extrapolate the method from the example. This can lead to confusion if the example does not address a particular instance of the method, leading to mass confusion over the ages.

2.2 The Evolution of the Gentle Art of Multidigit Multiplication

2.2.1 Multiplying with Roman Numerals

$CMLXXXII$	$XXXI$	982	31
$CDXCI$	$LXII$	491	62
$CCXLV$	$CXXIV$	245	124
$CXXII$	$\cancel{CCXLVIII}$	122	~~248~~
LXI	$CDXCVI$	61	496
XXX	$\cancel{CMXCII}$	30	~~992~~
XV	$MCMLXXXIV$	15	1984
VII	$MMMCMLXVIII$	7	3968
III	$\bar{V}CMXXXVI$	3	7936
I	$\bar{X}\bar{V}DCCCLXXII$	1	15872
	$\bar{X}\bar{X}\bar{X}CDXLII$		30442

Figure 2: $982 \times 31 = 30442$ *using Roman Numerals*

Consider first the method known to Fibonacci's contemporaries, who were still using Roman numerals, so that the problem then becomes CMLXXXII × XXXI. The Romans used a divide and conquer approach that had been adapted from the Egyptians, whose method was described by Ahmose in the Rhind Papyrus[23]. Like most early mathematicians, Ahmose did not give what we modernly consider a "method" for solving the problem; instead he presented an example and expected the student to adapt the example to problems as needed. I have expanded on the example and used the example to illustrate the method, where Ahmose would merely describe the example in detail, explaining what character to write where, but never explaining why.

In this approach the two numbers are written side by side at the top of two columns; and then on each row the number on the left is halved and the number on the right is

[22] The Rhind Papyrus dates back to BC 1650 and includes 84 mathematical problems and their solutions [Robins and Shute, 1987].

[23] J. J. O'Connor and E. F. Robertson of St Andrews University have provided a nice overview article at `http://www-groups.dcs.st-andrews.ac.uk/~history//HistTopics/Egyptian_papyri.html` of the mathematics included in the Rhind Papyrus, and Robins and Shute have written a lovely book about this papyrus [Robins and Shute, 1987].

doubled; both halving and doubling are two operations that are very easy to perform when using Roman numerals, especially if using a Roman counting board. After striking out all those numbers on the right whose left neighbour is even, the remaining numbers in the right-hand column are added together to calculate the final answer. As we can see from Figure 2 this will yield the correct answer, but whilst the intermediate steps are relatively simple, the resulting numbers are awkward to read or use. Furthermore, while both the Egyptians and Romans used this method, neither ever wrote down a convincing explanation of why this method works, in part because this method relies on binary arithmetic which was unknown at the time. 982, converted to binary, is 1111010110, and so we include the right hand values only when the left hand digit is a 1 (or odd)[24].

2.2.2 Fibonacci's "quadrilatero in forma scacherii"

2	4	5	0	0	0	7	
			4	3	2	1	
		3	0	2	4	7	7
		2	5	9	2	6	6
		2	1	6	0	5	5

Figure 3: Fibonacci's grid for the problem 4321×567 *[Sigler, 2002]*

In Fibonacci's method[25], which he names for squares on a chessboard – "quadrilatero in forma scacherii" in Italian – a grid is created whose width is one more than the number of digits in the larger number and whose height is equal to the number of digits in the smaller number. The example he gives is for 4321×567, and his illustration is shown in Figure 3; note that his answer is read off across the top, above 4321, and 567 is written from bottom up along the right hand side.

	9	8	2	
				1
				3

Figure 4: Fibonacci's starting setup.

In our example we will again be calculating 982×31; as per Fibonacci's instructions we will construct a grid that is four spaces wide and two spaces tall, and label the top with 982, and the right hand side with 31 writing the number from bottom to top, as we can see in Figure 4. The grid is then populated across from right to left, starting in the top right hand corner, and we must work from right to left and top to bottom so that we can maintain carries correctly.

[24] Since binary notation is thoroughly post-period, I will happily go into more detail in person but not, alas, in this paper.

[25] Chapter 3, second paragraph of *Liber Abaci*, from the translation by Sigler [Sigler, 2002]. Fibonacci uses the problem 4321×567, and specifically instructs where each number should be placed, but not why.

In each grid square we place the results of the multiplication of the digits of the greater and smaller number, which we have as column headers and row labels in modern terms. More specifically, we place the value of the ones digit, after adding in the carry – if any – from the previous grid square, and we carry the tens digit to the next square to the left. So to paraphrase Fibonacci[26], the 2 is multiplied by 1 so we write 2 in the topmost right square, and the 8 is also multiplied by the one, so we write 8. Lastly the 9 is multiplied by 1 so we write 9. Now on the next line the 2 is multiplied by 3 so we write 6, the 8 is multiplied by 3 which is 24 so we write 4 and carry the 2. Lastly the 9 is multiplied by 3 which is 27 but we must remember to add the 2 from the previous number so we write 9 and remember the 2. Then since there is no more multiplication on this line we write the final 2. While Figure 5 includes the carried digits in red, Fibonacci did not write them down but expected the student to remember them.

	9	8	2	
	9	8	2	1
$_{2}$2	$_{2}$9	4	6	3

Figure 5: Performing multi-digit multiplication with Fibonacci's approach. The carries, here indicated in red, are not written down but must be remembered by the person performing the calculation.

The final step is to read the answer from the grid. In this step, Fibonacci considers the diagonals of the grid, and the original numbers are ignored. To simplify matters, I have removed the original numbers from the outside of the grid so that we can see the diagonals clearly; Fibonacci neither explicitly describes them nor adds them to the diagram. To read off the final answer, the digits in each diagonal are summed, and the ones digit is written down while the carry digit, if any, is remembered. Again, I have added the carried digits in red; Fibonacci does not record them.

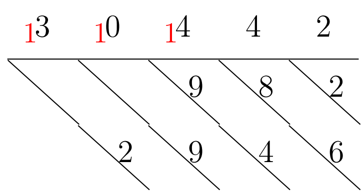


Figure 6: Reading off the answer from Fibonacci's grid with the original 982 *and* 31 *labels removed.*

In conclusion, then, Fibonacci's method is less cumbersome than the Roman method, but requires the student to remember to carry digits across from right to left as required, and to read diagonals while excluding the values in certain spaces in the middle of the sea of numbers, neither of which is easy to do quickly and accurately.

[26] Fibonacci's version is wordier than this paraphrase, by the way.

2.2.3 Pacioli's "gelosia"

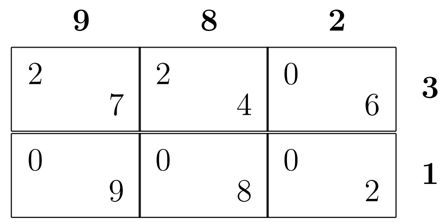


Figure 7: Setting up the original lattice.

In contrast, Pacioli's "gelosia" method, as described in his *Summa*, records the carry digits. Moreover, numbers are written from left to right and top to bottom, and the diagonals now slant down and to the left, rather than up and to the left. As a result, the interior grid squares can be calculated in any order, and the student does not have to worry about interruptions and forgotten carry digits. This habit of writing *everything* down becomes even more explicit when Pacioli turns his attention to bookkeeping.

The steps are fairly straightforward; first one number is written along the top of a table, one digit per column, and the other number is written to the right of the table, one digit per row. Then the table is filled with the results of single digit multiplication; each square contains the result of multiplying the column header with the row label. Thus, in the top left-most square we write the number 27, because $9 \times 3 = 27$. Figure 7 shows the completed table.

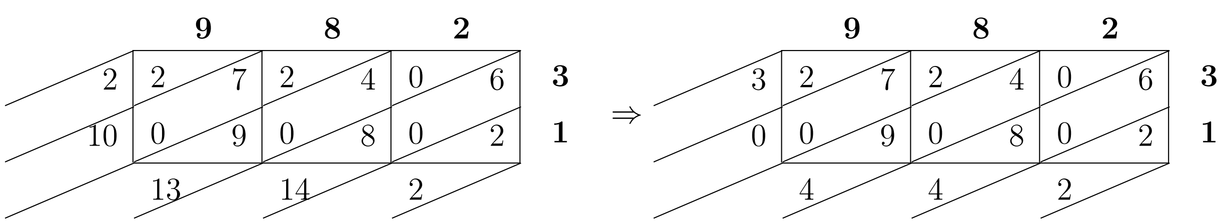
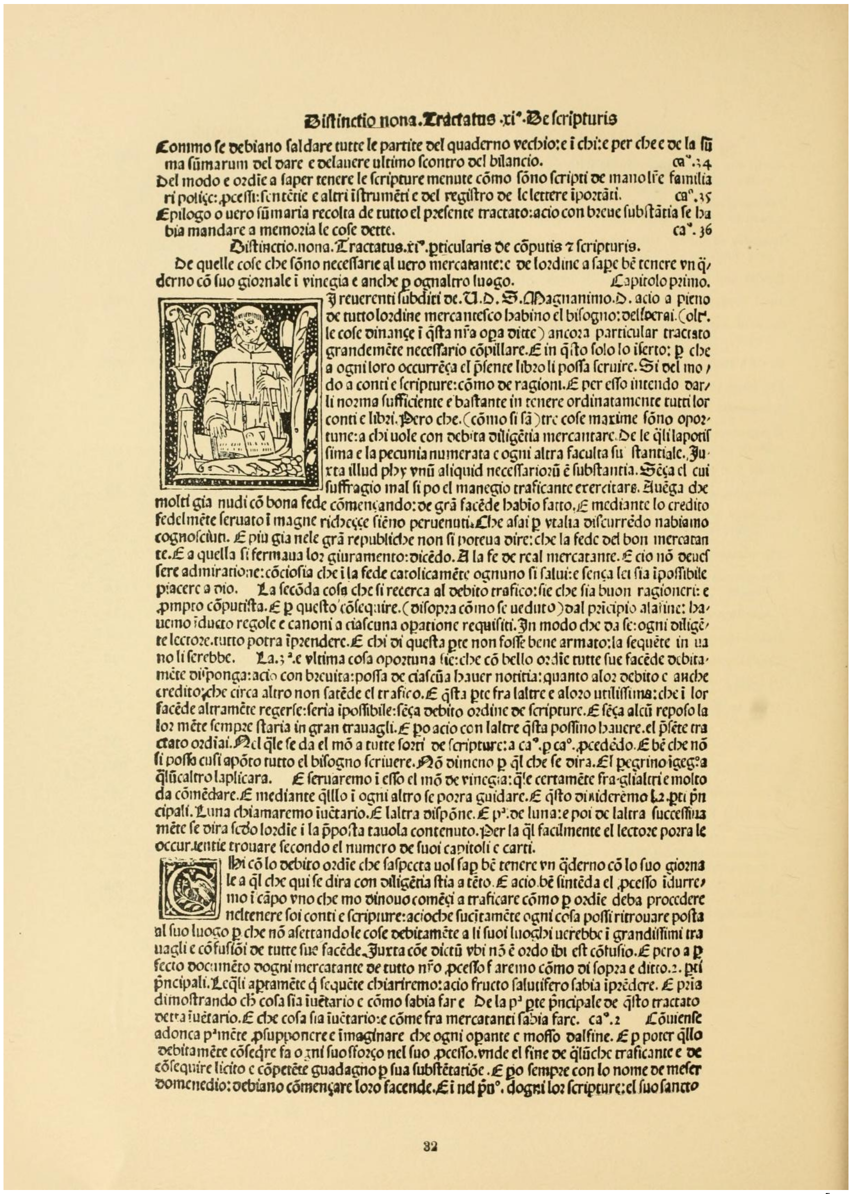


Figure 8: Summing the diagonals, before the carries are carried over to the next diagonal to the left, and then adding the carries to each diagonal shows that $982 \times 31 = 30442$.

Next, we carefully draw diagonals from top right to bottom left, and we sum the numbers inside each diagonal, as we see in Figure 8. Since some sums may add to a number greater than 10, we need to carry the left[27] digit to the next diagonal column leftwards. Finally we can read the answers around the left and bottom of the box. Unlike Fibonacci's method, there is no need to memorize extra digits since everything is written down, and unlike the Roman method, we don't have to worry about having to strike certain values out. Everything that is written down is used, and the answers read easily down the page and from left to right.

[27] In modern parlance, the 10's digit.

3 Pacioli and the Art of Double Entry Book Keeping



Distinctio nona. Tractatus xi°. De scripturis

Commo se debiano saldare tutte le partite del quaderno vecchio: e i chi: e per che e de la sūma sūmarum del dare e delauere ultimo scontro del bilancio. ca°.34

Del modo e ordie a saper tenere le scripture menute cōmo sōno scripti de mano lr̄e familiari poliçe: ꝓcessi: sentētie e altri istrumēti e del registro de le lettere ipoztāti. ca°.35

Epilogo o uero sūmaria recolta de tutto el presente tractato: acio con breue substātia se habia mandare a memoria le cose dette. ca°.36

Distinctio. nona. Tractatus. xi°. ꝑticularis de cōputis 7 scripturis.

De quelle cose che sōno necessarie al uero mercatante: e de lordine a sape bē tenere vn q̄derno cō suo giornale i vinegia e anche p ognaltro luogo. Capitolo primo.

I reuerenti subditi de. V. D. S. Magnanimo. D. acio a pieno de tutto lordine mercantesco habino el bisogno: delibera. (oltra le cose dinançe i q̄sta nr̄a opa ditte) ancora particular tractato grandemēte necessario cōpillare. E in q̄sto solo lo iserto: p che a ogni loro occurrēça el p̄sente libro li possa seruire. Si del modo a conti e scripture: cōmo de ragioni. E per esso intendo darli norma sufficiente e bastante in tenere ordinatamente tutti lor conti e libri. Pero che. (cōmo si sā) tre cose maxime sōno oportune: a chi uole con debita diligētia mercantare. De le q̄li la potissima e la pecunia numerata e ogni altra faculta sustantiale. Iuxta illud phy vnū aliquid necessariorū ē substantia. Sēça el cui suffragio mal si po el manegio traficante exercitare. Auēga che molti gia nudi cō bona fede cōmençando: de grā facēde habio fatto. E mediante lo credito fedelmēte seruato i magne richeçe sieno peruenuti. Che asai p ytalia discurrēdo nabiamo cognosciuti. E piu gia nele grā republiche non si poteua dire: che la fede del bon mercatante. E a quella si fermaua lor giuramento: dicēdo. A la fe de real mercatante. E cio nō deuesse admiratione: cōciosia che i la fede catolicamēte ognuno si salui: e sença lei sia ipossibile piacere a dio. La secōda cosa che si recerca al debito trafico: sie che sia buon ragioneri: e ꝓmpto cōputista. E p questo cōseguire. (di sopra cōmo se ueduto) dal pricipio alafine: hauemo iducto regole e canoni a ciascuna opatione requisiti. In modo che da se: ogni diligēte lectore. tutto potra iprendere. E chi di questa pte non fosse bene armato: la sequēte in uano li serebbe. La. 3ª. e vltima cosa oportuna sie: che cō bello ordie tutte sue facēde debitamēte disponga: acio con breuita: possa de ciascūa hauer notitia: quanto alor debito e anche credito: che circa altro non satēde el trafico. E q̄sta pte fra laltre e aloro utilissima: che i lor facēde altramēte regerse: seria ipossibile: sēça debito ordine de scripture. E sēça alcū reposo la lor mēte sempre staria in gran trauagli. E p o acio con laltre q̄sta possino hauere. el p̄sēte tractato ordiai. Nel q̄le se da el mō a tutte sorti de scripture: a ca°. p ca°. ꝓcedēdo. E bē che nō si possa cusi apōto tutto el bisogno scriuere. Nō dimeno p q̄l che se dira. El pegrino igegno a q̄lūcaltro laplicara. E seruaremo i esso el mō de vinegia: q̄le certamēte fra glialtri e molto da cōmēdare. E mediante q̄llo i ogni altro se porra guidare. E q̄sto dividerēmo i 2. pti p̄ncipali. Luna chiamaremo iuctario. E laltra dispōne. E p ª. de luna: e poi de laltra successiuamēte se dira scōdo lordie i la p̄posta tauola contenuto. Per la q̄l facilmente el lectore porra le occurentie trouare secondo el numero de suoi capitoli e carti.

Chi cō lo debito ordie che saspecta uol sap bē tenere vn q̄derno cō lo suo giornale a q̄l che qui se dira con diligētia stia a tēto. E acio bē sintēda el ꝓcesso idurremo i cāpo vno che mo di nouo comēçi a traficare cōmo p ordie deba procedere nel tenere soi conti e scripture: acioche sucitamēte ogni cosa possi ritrouare posta al suo luogo p che nō asettando le cose debitamēte a li suoi luoghi uerebbe i grandissimi trauagli e cōfusiōi de tutte sue facēde. Iuxta cōe dictū vbi nō ē ordo ibi est cōfusio. E pero a p fecto documēto dogni mercatante de tutto nr̄o ꝓcesso faremo cōmo di sopra e ditto. 2. pti p̄ncipali. Le q̄li aptamēte q̄ sequēte chiariremo: acio fructo salutifero sabia iprēdere. E prima dimostrando che cosa sia iuētario e cōmo sabia fare. De la pª pte p̄ncipale de q̄sto tractato detta iuētario. E che cosa sia iuctario: e cōme fra mercatanti sabia fare. ca°.2 Cōuiense adonca pªmēte ꝓsupponere e imaginare che ogni opante e mosso dal fine. E p poter q̄llo debitamēte cōseq̄re fa ogni suo sforço nel suo ꝓcesso. vnde el fine de q̄lūche traficante e de cōsequire licito e cōpetēte guadagno p sua substētatiōe. E po sempre con lo nome de meser domenedio: debiano cōmençare loro facende. E i nel p̄n°. dogni lor scripture: el suo sancto

32

Figure 9: *The opening section of Pacioli's treatise on double entry book keeping[Geijsbeek, 1914].*

Pacioli's text *Summa de arithmetica, geometria, proportioni et proportionalita* is clearly a textbook intended for teaching (probably) young students the arts of mercantile math. Unlike his predecessor Fibonacci, Pacioli writes his book in the vernacular Italian he also uses when teaching students. While *Summa* is not his first textbook – he wrote several shorter versions while working as a mathematics teacher – it is certainly his most published book[28].

We turn our attention now the most interesting section of his treatise, which describes the art of bookkeeping. Pacioli begins his treatise on double entry book keeping with the following chapter 1. We will not quote the entire translation of the treatise[29], but here Pacioli justifies both why the book keeping was considered so important, and why it is important for merchants to manage their journals and ledgers in an orderly way. Notice how informal the text is; Pacioli is speaking directly to his reader and trying to convince them that the methodology he presents has value. Based on the widespread adoption of his views, I would argue that Pacioli's reasoning was considered good.

> Chapter 1
>
> Things that are necessary to the good merchant and the method of keeping a ledger with its journal, in Venice and Elsewhere.
>
> *In order that the subjects of His Illustrious Highness, the most honorable and magnanimous Duke of Urbino (D. U. D. S. – Docis Urbini Domini Serenissimi), may have all the rules that a good merchant needs, I decided to compile, in addition to the subjects already treated in this work, a special treatise which is much needed. I have compiled it for this purpose only, i.e., that they (the subjects) may whenever necessary find in it everything with regard to accounts and their keeping. And thereby I wish to give them enough rules to enable them to keep all their accounts and books in an orderly way. For, as we know, there are three things needed by any one who wishes to carry on business carefully. The most important of these is cash or any equivalent, according to that saying, Unum aliquid necessarium est substantia [One thing is the necessary substance]. Without this, business can hardly be carried on.*
>
> *It has happened that many without capital of their own but whose credit was good, carried on big transactions and by means of their credit, which they faithfully kept, became very wealthy. We became acquainted with many of these throughout Italy. In the great republics nothing was considered superior to the word of the good merchant, and oaths were taken on the word of a good merchant. On this*

[28] While no accurate records of the original print run have survived, it was reprinted multiple times. Sangster considers the printing of the *Summa* and concludes that the initial print run in 1494 could easily consist of 2000 copies, a total which does not include the reprints or the second edition [Sangster, 2007].

[29] Pacioli includes 36 chapters, including case studies and examples.

confidence rested the faith they had in the trustworthiness of an upright merchant. And this is not strange, because, according to the Christian religion, we are saved by faith, and without it it is impossible to please God.

The second thing necessary in business is to be a good bookkeeper and ready mathematician. To become such we have given above (in the foregoing sections of the book) the rules and canons necessary to each transaction, so that any diligent reader can understand it all by himself. If one has not understood this first part well, it will be useless for him to read the following.

The third and last thing is to arrange all the transactions in such a systematic way that one may understand each one of them at a glance, i. e., by the debit (debito – owed to) and credit (credito – owed by) method. This is very essential to merchants, because, without making the entries systematically it would be impossible to conduct their business, for they would have no rest and their minds would always be troubled. For this purpose I have written this treatise, in which, step by step, the method is given of making all sorts of entries. Although one cannot write out every essential detail for all cases, nevertheless a careful mind will be able, from what is given, to make the application to any particular case.

This treatise will adopt the system used in Venice, which is certainly to be recommended above all the others, for by means of this, one can find his way in any other. We shall divide this treatise in two principal parts. The one we shall call the Inventory, and the other, Disposition (arrangement). We shall talk first of the one and then of the other, according to the order contained in the accompanying Table of Contents, from which the reader may take what he needs in his special case.

He who wants to know how to keep a ledger and its journal in due order must pay strict attention to what I shall say. To understand the procedure well, we will take the case of one who is just starting in business, and tell how he must proceed in keeping his accounts and books so that at a glance he may find each thing in its place. For, if he does not put each thing in its own place, he will find himself in great trouble and confusion as to all his affairs, according to the familiar saying, Ubi non est ordo, ibi est confusio (Where there is no order, there is confusion). In order to give a perfect model to every merchant, we will divide the whole system, as we have said, in two principal parts, and we will arrange

> *these so clearly that one can get good results from them. First, we will describe what the inventory is and how to make it*[30].

What follows is a brief overview of Pacioli's writings in his treatise, based on the translation by Geijsbeek.

3.1 The Parts of the System

Pacioli divides the system of keeping mercantile books into two parts; the inventory, including both a listing of the goods in stock and a list of debits owed and credits due, and then the journal and the ledger. Implicit in Pacioli's description is that the inventory should be taken once, at the beginning of the mercantile venture. It consists of a list of the merchants belongings, or as Pacioli puts it "whatever he has in this world, personal property or real estate"[31]. The belongings are to be listed in order, beginning with those that are most valuable *and* most likely to be lost, so that jewellery, for instance, is listed above real estate[32]. He does not mention debits and credits directly, but offers a sample inventory in his third chapter, which includes as the final items in the inventory a list of moneys on deposit with banks, a list of debtors who owe the merchant money, and a list of creditors to whom the merchant in turn owes money. It is recommended that the entire inventory is recorded in one day, so as to avoid muddying the waters of commerce.

The inventory having been completed, the merchant is now ready to step into business. First a diversion in Chapter 4, encouraging the merchant to keep good books and write down absolutely everything of importance so as to never lose his riches. And his[33] first order of business was to purchase three books to keep the accounts. These three books are in turn the Memorandum (or the Memoriale), the Journal (or the Giornale), and the Ledger (or the Quaderno).

The Memorandum book, sometimes known as the scrap book or blotter, is the daily journal of every transaction that happens in the order it occurs. Each transaction will

[30] Translation by Geijsbeek[Geijsbeek, 1914]. The original was written by Pacioli in the vernacular, rather than in Latin, so as to be accessible to his audience – the merchants of Italy.

[31] Translation of Chapter 2 by Geijsbeek[Geijsbeek, 1914].

[32] To quote Pacioli, "for the real estate, such as houses, lands, lakes, meadows, ponds, etc., cannot be lost as personal property."[Geijsbeek, 1914], which strongly suggests that Pacioli is not allowing for the presence of politics in the slightest.

[33] Sadly, this being Italy and not the Low Lands, now known as the Netherlands, merchants were invariably male.

include full details, including who was involved, what was exchanged[34], and what promises were made. At the same time, the Memorandum is the book used by all who work for the merchant, be they as barely literate as they may be, and so the format of the Memorandum is not assumed to be pristinely perfect. It will be the task of the bookkeeper to take what is in the Memorandum and transcribe it properly into the Journal.

Next comes the Journal. Much is made by Pacioli of the need to mark the Memorandum and Journal books, so that each can be quickly recognized, and he admonishes the merchant to mark the first book of each by the Holy Cross, and then by the letters of the alphabet, so that it is easy to place the books in order. It is important, as he explains, that both the books be marked as to their order – so that a missing book might quickly be detected – and then the pages inside be marked in order. Since each entry includes a date, this could be used easily to order the pages but, in the event that a merchant completed too many transactions to fit on one page, a page might be torn out of the ledger leaving the fraud undetectable as far as the use of dates was concerned. Hence the first task of any merchant in acquiring any new book is to immediately number all the pages. Moreover, as soon as the books have been acquired and the pages numbered, the merchant is expected to write, in his own handwriting, on the front page what kind of money will be used in recording the transactions. These books then are taken to the mercantile officer[35] who records the merchant's name, the markings on the books, how the books are names and how many pages they have, etc, and then writes the officer's name and attaches the seal of that office. Thus the cities tracked their merchants.

Pacioli spends several chapters discussing how to enter transactions in the Memorandum and the Journal. The first step is to refer back to the Inventory, which is copied out nicely into both; note that *everything* is recorded, and always in duplicate. The emphasis here is on faithful recording, for it is in these books that the merchant records *and provides proof*[36] of his wealth and belongings. In fact, Pacioli explicitly details how to track the copying of an entry from the Memorandum to the Journal; in the Memorandum's entry a horizontal line is drawn from the last word of the description to the column of figures. When the entry

[34] One of the ways Pacioli demonstrates his genius is by describing things in general terms. Fibonacci might have listed each possible type of transaction and what should have been recorded for that transaction, whereas Pacioli simplifies by explaining *why* everything is recorded, and then leaves it to the merchant to decide what "everything" includes. Just as in multiplication, Pacioli is generalizing, and only then giving specific examples.

[35] The Mercantile Office in each city tracked the merchants and inspected their account books.

[36] And lo, it came to pass that you owned only that which was written in your ledger, or the ledger of your master, and the bureaucrats put upon the ledger their seal, and all was good.

is transcribed into the Journal, a diagonal line is added to the horizontal line[37].

The third and final book is the Ledger. Pacioli suggests that the Ledger should have at least twice as many pages as the Memorandum and Journal, in part because every entry in the Journal will require two entries in the Ledger; one in the debit and and one in the credit, which were indicated in the Journal by "Per" for the debtor and "A" for the creditor. Moreover, since the two entries are not usually recorded on the same page, each debit entry should include the page number for the credit entry. Then if all the entries are recorded faithfully and correctly, the total sum of the debits will always exactly equal the total sum of the credits[38]. Entries are recorded on different pages because each debit or credit account gets its own section, which may span from half a page to multiple pages as required, and Pacioli emphasizes how important it is to keep an index – preferably alphabetic – for the Ledger.

In Chapters 17 and 18, Pacioli explicitly explains how to keep the books, and record transactions, as a merchant in Venice, dealing with the Camera de L'Impresti[39] and the office of the Venetian Messetaria[40]. He particularly comments on the turnover in the clerical staff at the bank, and how the merchant should keep his own books with perfectly accuracy to offset the errors that may occur when one clerk replaces another.

The remaining chapters cover the technicalities of book keeping for the household, the partnership, and when traveling abroad[41], before finally concluding with the closing of the old ledgers, either at the end of the year or after the ledger is full.

Curiously, Chapter 35 is titled "How and in what order papers should be kept, such as manuscripts, family letters, policies, processes, judgements and other instruments of writing and the record book of important letters"[42] and contains instructions both on how to address and how to store a variety of letters. In this chapter, he explains how to record important letters – record a copy of the letter in one's Record Book of Important Letters – as well as the matter of addressing them. He recommends, for instance, marking the outside of the sealed envelope with "your special mark, so that they may know that it is correspondence of a merchant, because great attention is given to merchants, for they are the ones, as we said at the beginning of this

[37] From Chapter 12[Geijsbeek, 1914]; Pacioli's description is detailed and emphasizes that the merchant should follow common mercantile custom.

[38] This is the *key* concept of double entry bookkeeping.

[39] The Municipal Loan Bank managed by the Sestieri, or districts.

[40] The exchange in Venice, which allows you to buy merchandise through brokers.

[41] On travels, take a small journal and ledger, and reconcile them after returning.

[42] Geijsbeek, again[Geijsbeek, 1914].

treatise, who support our republics"[43] which strongly resembled the address we commonly write on the top left corner of addressed envelopes nowadays.

4 The Aftermath

Pacioli was many things, but he was above all a master teacher. Born circa 1447 in Sansepolcro, he was taken in[44] as a young boy by the family of the local merchant Folco de' Belfolci. He studied in the local abbaco[45], or school, and moved in 1464 to Venice to become an abbaco teacher himself and continue his studies in mathematics. It was in Venice, as the tutor to three sons of the the wealthy fur merchant Ser Antonion de Rompiasi, that he learned how to keep mercantile books, and the treatise Pacioli published in 1494 emphasized in every way the Venetian approach to keeping books, while allowing that merchants in other cities might have slightly different ways. After the teenage sons had grown to adulthood, he spent the remainder of his life as an itinerant teacher; he spent time in Rome in the early 1470s before leaving for Naples and then Perugia. Travel in those days was dangerous, although monks could usually travel freely. It may be possible that these dangers played a part in Pacioli's taking the vows of a Franciscan friar, when he joined the Conventual Franciscans before 1475; he was allowed and encouraged to travel widely and continue studying and teaching mathematics.

So in 1494, his magnum opus completed, Pacioli returned to Venice. His work was printed by Paganino de Paganini, and funded by his patron Marco Sanuto. The volume was priced at 119 soldi[46] and measured 25cm by 30 cm[47], with 615 densely printed pages. This was not a slim texbook; it was a heft mathematical tome, written in the vernacular of the time. After publication of the book, Pacioli was summoned to Milan by Ludovico Sforza, to take up Milan's first Chair of Mathematics. Pacioli was brought to the attention of Sforza by Leonardo da Vinci who collaborated with Pacioli for the duration of Pacioli's visit[48]. Tracing out his travels, he rarely stayed in any one location for more than a few years, and

[43] Geijsbeek, Chapter 35[Geijsbeek, 1914].

[44] The record is unclear as to whether he was adopted, apprenticed, or exchanging his labour for room and board. Based on his later accomplishments he did attend a local abbaco and mastered mathematics well enough to find work as a tutor.

[45] The teachers of these schools were 'abbacists" and based their teachings on Fibonacci's "Liber Abaci", which was written in latin[Gleeson-White, 2011].

[46] In comparison, the popular "Aesops Fables" was price at 2 soldi[Gleeson-White, 2011].

[47] 25cm by 30 cm is roughly 9.8" by 11.8".

[48] The visit was cut short when Milan was invaded by Louis XII of France.

his final posting was once more as a university mathematics professor in Rome, where he died in 1517.

Throughout Pacioli's life he published mathematics textbooks, all written in vernacular Italian, and often intended for merchants and engineers who sought practical methods. One of the joys of reading his words – albeit in translation – is that he sought out the underlying method and explained it first, rather than leaving the method to be discerned by the reader as Ahmose, Euclid, and Fibonacci did. So whilst Pacioli was reporting on mercantile bookkeeping methods that were already in use in Venice, he explained them in such a way that merchants across the known world could adapt them to local circumstances, and so we still use the same – or very similar – techniques today.

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