



Figure 1: STARTING SETUP OF THE BOARD.

## 1 Introduction

Rithmomachy – sometimes known as the Philosophers’ Game – was developed in the 11th century, and played well into the seventeenth century. It was particularly popular at monastery schools where it served a dual purpose of drilling students in their mathematical facts, as well as providing an example of the contemplative studies of the mathematical philosophy of the sixth century philosopher Boetthius, who in his “de Arithmetica” studied the properties of numbers, and modular arithmetic<sup>1</sup>. The game requires the ability of its players to decompose numbers into factors, or alternatively to multiply and add quickly and accurately<sup>2</sup>.

## 2 Setting up the Board

The pieces consist of circles, triangles, and squares, each marked with a number and a colour, some of which are combined together to form the player’s pyramid. Some numbers repeat; black, for instance, has 2 round 9’s and a triangular 9.

The board is set up between the two players Black and White. Each piece is played according to its shape and number; the pyramid counts as a hybrid shape and number – as long as it contains at least one piece of a certain shape, it can act as that shape and number; it also acts as the sum of all the numbers remaining in the pyramid.

The pieces are placed as in Figure 1;

- the black pyramid consists of two squares (64 and 49), two triangles (36 and 25) and a circle (16), all of which together add up to the pyramid’s value of 190.

<sup>1</sup>And other things too, but we’re simplifying here.

<sup>2</sup>Perhaps we should follow this class up with a class on medieval calculators, including abacii and Napier’s bones. Calculators may be handy, but they are so excessively non-period.

- the white pyramid consists of two squares (36 and 25), two triangles (16 and 9) and two circles (4 and 1), all of which together add up to the pyramid's value of 91.

## 3 Moving and Capturing

### 3.1 Moving

Pieces must travel in a straight line horizontally, vertically, or diagonally, cannot jump over other pieces, and must land on an empty space. The number of steps a piece can move is controlled by its shape:

- circle: one step (and has one side)
- triangle: three step (and has three sides)
- square: four step (and has four sides)

A pyramid can move as though it had the shape of *any* of its component pieces. Thus, the starting pyramid can move either 1, 3, or 4 steps in any direction . . . once the pyramid is no longer hemmed in by its neighbours. Until then it can only move one step diagonally backwards.

### 3.2 Capturing

There are four different ways to capture a piece. Some captures rely on movement, others on arithmetic properties, and yet others on position.

- **by encounter:** an encounter occurs when a piece lands on a square containing an enemy piece. The player *never actually moves* his piece, but simply removes his opponent's piece from the board.

This kind of capture depends only on the mobility of the attacking piece.

- **by attack:** this attack requires multiplication. The attacking piece, which has value  $x$ , must have a lower value than the defending piece, with value  $y$ . Moreover, the line between the two pieces must not contain any other pieces. Then, if the value of the attacker  $x$  times the number of blank squares between the two pieces  $d$  is equal to  $y$ , then the defender is defeated. (I.e. if  $x \times d = y$ , then  $y$  loses.) Again, the attacking piece does not move.

This kind of capture depends only on the values of the defending and attacking pieces, and the distance between them. It does *not* depend on the usual mobility of the attacking piece.

- **by ambush:** this attack requires positioning and addition. A defending piece is ambushed if it is boxed in by two attacking pieces, one to either side in a straight line. (Boxing in can happen horizontally, vertically, or diagonally.) The ambush is successful if the sum of the attacking pieces matches the value of the defending piece.

This kind of capture depends on the values of the defending and attacking pieces, and requires the move of an attacking piece. (The second attacker must already be in position.)

- **by siege:** this attack requires positioning. To capture a defending piece by siege, four attackers must surround it horizontally and vertically. Diagonally adjacent pieces do not affect the siege. (If a piece is on the edge of the board, it need only be surrounded on three sides for a successful siege.)

This kind of capture depends only on the positioning of the attacking pieces, and will usually require multiple turns to set up, since four pieces need to move into position.

### 3.2.1 The Pyramid

The pyramid is ruled by several special rules, in part because these pieces are so powerful.

When attacking, a pyramid can act as any of the pieces it contains. Thus it can at first move 1, 3, or 4 steps, and if attacking by attack or ambush, can use the value of any one of the pieces it contains.

Pyramids cannot be captured if they have not been breached (i.e. they still contain all the pieces they started with.) Before capturing an entire pyramid, the player must first capture the bottom piece, which is either the white 36 or the black 64.

Once the pyramid has been breached, it can be captured by capturing any one of its component parts. However, a defending player is entitled to offer a ransom to the attacker when his pyramid is threatened, consisting of a piece still on the board with the same value. If the ransom is accepted, the attacker may accept the alternate piece. If not, the attacker wins the entire pyramid.

## 4 Victory Conditions

The first step, besides setting up a board, is deciding on which victory condition to use. Here we will discuss the first five options, each of which is considered a “common” victory. In each case, the objective of the game is to capture the opponents’ pieces. However, each of the five victory conditions determines which and how many pieces need to be captured.

- **Corpore:** depends solely on the number of pieces captured. Players must agree that victory will go to the first player who captures the agreed upon number of pieces. (This is a good condition for beginners; for a quick game, the number would be 5, say.)
- **Bonis:** depends on the value of the pieces. Players agree on the victory sum; victory is attained when one player captures enough pieces to reach or surpass the agreed upon sum.
- **Lite:** depends on the value of the pieces, as well as the digits. (I.e. 4 has 1 digit, 64 has 2 digits, 289 has 3.) Players agree on both the final sum and the number of digits before the start of the game. Usually the sum of the captured pieces must reach or exceed the sum, but must be exactly the number of digits.
- **Honore:** depends on the value and number of the pieces. Players agree on a final sum, and on the number of pieces. To win, a player must capture a set number of pieces, whose sum must equal or exceed the agreed upon sum.
- **Honore Liteque:** depends on the value and digits of the pieces, as well as the number of pieces. This is essentially the combination of Lite and Honore; players agree beforehand on the final sum, the number of digits used, and the number of pieces used.

Note that players may capture pieces which they choose not to use in meeting the victory condition(s). Thus, playing Honore Liteque, capturing an extra piece is not fatal.